Generalizations of the Andrews-Alder Theorem in Partition Theory

Partitions with parts that differ by at least d, called d-distinct partitions, arise in the famous Euler's identity and first Rogers-Ramanujan identity. These identities state that the number of 1-distinct or 2-distinct partitions of n is equal to the number of partitions of n into parts that are ± 1 modulo 4 or 5, respectively. Alder showed that for $d \geq 3$ no identity of such a type exists, and conjectured what is now the Andrews-Alder Theorem, namely that there are at least as many d-distinct partitions of n as partitions of n into parts that are ± 1 modulo $d+3$. This was proved partially by Andrews in 1971, by Yee in 2008, and was fully resolved by Alfes, Jameson and Lemke Oliver in 2011.

In 2020, Kang and Park constructed an extension of Alder's conjecture which relates to the second Rogers-Ramanujan identity. Namely, that there are at least as many d-distinct partitions of n with parts at least 2 as partitions of n into parts that are ± 2 modulo $d+3$, excluding the part $d+1$. Kang and Park proved their conjecture for the cases when $d = 2^r - 2$ and n is even.

Here, we prove Kang and Park's conjecture for all $d \geq 62$. Toward proving the remaining cases, we adapt work of Alfes, Jameson and Lemke Oliver to generate asymptotics for the related functions. Additionally, we present a more generalized infinite family of conjectures and provide proofs for infinite classes of n and d.

This work is joint with REU students Adriana Duncan (Tulane), Simran Khunger (Carnegie Mellon), and Ryan Tamura (Berkeley) and was partially supported by the National Science Foundation REU Site Grant DMS-1757995, and Oregon State University.